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$\Delta S = 0$ effective weak chiral Lagrangian from the instanton vacuum

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Abstract. We investigate the $\Delta S = 0$ effective chiral Lagrangian from the instanton vacuum. Based on the $\Delta S = 0$ effective weak Hamiltonian from the operator product expansion and renormalization group equations, we derive the strangeness-conserving effective weak chiral Lagrangian from the instanton vacuum to order $\mathcal{O}(p^2)$ and the next-to-leading order in the $1/N_c$ expansion at the quark level. We find that the quark condensate and a dynamical term which arise from the QCD and electroweak penguin operators appear in the next-to-leading order in the $1/N_c$ expansion for the $\Delta S = 0$ effective weak chiral Lagrangian, while they are in the leading order terms in the $\Delta S = 1$ case. Three different types of form factors are employed and we find that the dependence on the different choices of the form factor is rather insensitive. The low-energy constants of the Gasser–Leutwyler type are determined and discussed in the chiral limit.

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1 Introduction

A great deal of attention has been paid to parity violation (PV) in the electroweak standard model (SM) well over decades in the context of high-precision tests for the SM [1]. A recent series of parity-violating experiments in atomic physics measured the weak charge of the SM [2–5]. Its discrepancy with the SM implies a possibility of new physics, for example, a possible existence of the Z' boson in addition to the Z^0 boson [6–8]. Recently, strangeness-conserving ($\Delta S = 0$) weak processes have paved the way of probing subtle properties of the nucleon such as the strangeness in the nucleon: The strange vector form factors were recently extracted by measuring the asymmetries of PV $\boldsymbol{ep}$ parity-violating scattering [9]. Hadronic and nuclear PV processes, however, are far from being clearly understandood due to the screening of the strong interaction.

A simple framework to describe hadronic and nuclear PV processes is one-boson exchange (OBE) such as π-, ρ-, and ω-changes [10–12] à la the strong nucleon–nucleon potential through OBE [13, 14]. The main ingredients of the PV OBE model are the weak meson–nucleon coupling constants such as h_π, h_ρ, and h_ω, which can be extracted from PV observables in various hadronic and nuclear reactions like $\boldsymbol{pp}$ elastic scattering [15], $\boldsymbol{n}p \to d\gamma$ [16, 17], and $^{18}\mathrm{F}^* \to {}^{18}\mathrm{F}$ [18]. In particular, the weak pion–nucleon coupling constant h_π^1 is one of the most important quantities dominant in PV weak hadronic processes at low-energy regions [10,19–21]. However, disagreement in determining the h_π^1 still exists [22] theoretically as well as experimentally.

A recent series of works [23–25] studied the h_π^1 within the Skyrme model, based on the effective current–current interaction which can be identified as a factorization scheme. However, it is natural to describe the $\Delta S = 0$ PV processes based on the effective weak Hamiltonian evolved from a scale of 80 GeV down to around 1 GeV [10,26–29]. Furthermore, it is well known that the non-leptonic weak processes defy any explanation from the factorization, or the strict large-N_c limit. The octet enhancement in $K \to \pi\pi$ decays is partially explained by gluon penguin diagrams, which indicates that the strong interaction plays an essential role in describing the non-leptonic or hadronic decay processes. Thus, in the present work, we shall derive the $\Delta S = 0$ effective weak chiral Lagrangian (EWχL) incorporating the effective weak Hamiltonian [10], based on the non-local chiral quark model from the instanton vacuum [30], which will provide a good theoretical framework in studying the weak coupling constants [31]. We shall consider the $\Delta S = 0$ EWχL to order $\mathcal{O}(p^2)$ in the chiral limit and to the next-to-leading order (NLO) in the $1/N_c$ expansion, keeping in mind that the present results of the NLO in N_c corrections are just a part of the whole $1/N_c$ NLO contributions.

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Table 1. Strong enhancements with selected values of K. $\theta_{\rm W}$ and $\theta_{\rm C}$ are the Weinberg and the Cabbibo angles, respectively

	$K=1$	$K=4$	$K=7$
α_{11}	$\cot\theta_{\rm C}$	$1.126\cot\theta_{\rm C}$	$1.266\cot\theta_{\rm C}$
α_{22}	$\tan\theta_{\rm C}$	$1.126\tan\theta_{\rm C}$	$1.266\tan\theta_{\rm C}$
β_{11}	0	$-0.307\cot\theta_{\rm C}$	$-0.479\cot\theta_{\rm C}$
β_{22}	0	$-0.307\tan\theta_{\rm C}$	$-0.479\tan\theta_{\rm C}$
γ_{11}	$-0.002\left(1-\frac{2}{3}\sin^2\theta_{\rm W}\right)\csc 2\theta_{\rm C}$	$0.077\left(1-\frac{2}{3}\sin^2\theta_{\rm W}\right)\csc 2\theta_{\rm C}$	$0.163\left(1-\frac{2}{3}\sin^2\theta_{\rm W}\right)\csc 2\theta_{\rm C}$
γ_{12}	$0.007\sin^2\theta_{\rm W}\csc 2\theta_{\rm C}$	$0.001\sin^2\theta_{\rm W}\csc 2\theta_{\rm C}$	$0.006\sin^2\theta_{\rm W}\csc 2\theta_{\rm C}$
γ_{21}	$-0.671\sin^2\theta_{\rm W}\csc 2\theta_{\rm C}$	$-0.772\sin^2\theta_{\rm W}\csc 2\theta_{\rm C}$	$-0.898\sin^2\theta_{\rm W}\csc 2\theta_{\rm C}$
γ_{22}	$(1-2\sin^2\theta_{\rm W})\csc 2\theta_{\rm C}$	$1.101(1-2\sin^2\theta_{\rm W})\csc 2\theta_{\rm C}$	$1.236(1-2\sin^2\theta_{\rm W})\csc 2\theta_{\rm C}$
ρ_{11}	0	$-0.190\left(1-\frac{2}{3}\sin^2\theta_{\rm W}\right)\csc 2\theta_{\rm C}$	$-0.296\left(1-\frac{2}{3}\sin^2\theta_{\rm W}\right)\csc 2\theta_{\rm C}$
ρ_{12}	$0.003\sin^2\theta_{\rm W}\csc 2\theta_{\rm C}$	$0.260\sin^2\theta_{\rm W}\csc 2\theta_{\rm C}$	$0.453\sin^2\theta_{\rm W}\csc 2\theta_{\rm C}$
ρ_{21}	$-0.001\sin^2\theta_{\rm W}\csc 2\theta_{\rm C}$	$0.002\sin^2\theta_{\rm W}\csc 2\theta_{\rm C}$	$-0.032\sin^2\theta_{\rm W}\csc 2\theta_{\rm C}$
ρ_{22}	0	$-0.307(1-2\sin^2\theta_{\rm W})\csc 2\theta_{\rm C}$	$-0.479(1-2\sin^2\theta_{\rm W})\csc 2\theta_{\rm C}$

The non-local chiral quark model induced from the instanton vacuum has several virtues: It was shown that this momentum dependence gives the correct end-point behavior of the quark virtuality for the pion wave function [32, 33]. Similarly, recent investigations on the effective weak chiral Lagrangian [34–36] indicate that the momentum-dependent quark mass plays a significant role in enhancing the $\Delta T=1/2$ channel. Furthermore, nonlocality of the quark introduces a unique feature to the low-energy constants (LEC) [37], compared to other models.

The paper is organized as follows: In Sect. 2, we show how to incorporate the $\Delta S=0$ effective weak Hamiltonian. In Sect. 3, we discuss the present results of the low-energy constants. In particular, the behavior of the LEC is studied with respect to the momentum-dependent quark mass. In the last section, we summarize the present work and draw conclusions.

2 $\Delta S=0$ effective weak chiral Lagrangian

In this section, we will show how to incorporate the $\Delta S=0$ effective weak Hamiltonian into the effective chiral action from the instanton vacuum. The detailed description can be found in [36]. We employ the $\Delta S=0$ effective PV weak Hamiltonian derived in [10]. The Hamiltonian reads

$$\mathcal{H}_W^{\Delta S=0}=\frac{G_{\rm F}}{\sqrt{2}}\cos\theta_{\rm C}\sin\theta_{\rm C}\times\left[\sum_{i,j=1}^{2}\left(\alpha_{ij}\mathcal{O}\left(A_i^\dagger,A_j\right)+\beta_{ij}\mathcal{O}\left(A_i^\dagger t_A,A_jt_A\right)+\text{h.c.}\right)+\sum_{i,j=1}^{2}\left(\gamma_{ij}\mathcal{O}\left(B_i^\dagger,B_j\right)+\rho_{ij}\mathcal{O}\left(B_i^\dagger t_A,B_jt_A\right)\right)\right],\quad(1)$$

where the operator $\mathcal{O}(M,N)$ is defined as $\mathcal{O}(M,N)\equiv-\psi^\dagger\gamma_\mu\gamma_5M\psi\psi^\dagger\gamma^\mu N\psi$ in Euclidean space, and t_A denotes the generator of the color $SU(3)$ group, normalized as $\mathrm{tr}\,t_At_B=2\delta_{AB}$. The definitions of the matrices A_i and B_i, and the coefficients α, β, γ and ρ are given in [10]. These coefficients are the functions of the scale-dependent Wilson coefficient $K(\mu)$ defined as

$$K(\mu)\equiv\left(1+\frac{g^2(\mu^2)}{16\pi^2}b\ln\frac{M_W^2}{\mu^2}\right),\quad(2)$$

where g is the strong coupling constant, μ is the renormalization point and specifies the mass scale, $b=11-2N_f/3$, and M_W is the mass of the W boson. K encodes the effect of the strong interaction from the perturbative gluon exchanges. Numerical values of the coefficients relevant to our discussion with various K values are listed in Table 1. We denote the four-quark operators generically by

$$\mathcal{Q}^i(x)=-\psi^\dagger(x)\Gamma_1^i\psi(x)\psi^\dagger(x)\Gamma_2^i\psi(x)\,,\quad(3)$$

where $i=1\dots12$ labels each four-quark operator in the effective weak Hamiltonian and $\Gamma_{1(2)}^i$ consist of the γ and the flavor matrices. Thus, the effective weak Hamiltonian can be rewritten as follows:

$$\mathcal{H}_W^{\Delta S=0}=\sum_{i=1}^{12}\mathcal{C}_i\mathcal{Q}^i(x),\quad(4)$$

where $\mathcal{C}_i$ denotes α, β, γ and ρ in (1).

The $\Delta S=0$ effective PV weak Hamiltonian can be incorporated into the non-local chiral quark model as follows:

$$\exp(-S_{\rm eff}^{\Delta S=0})=\int\mathcal{D}\psi\mathcal{D}\psi^\dagger\exp\left(\int\mathrm{d}^4x\left(\psi^\dagger D\psi-\mathcal{H}_W^{\Delta S=0}\right)\right),\quad(5)$$

where the D denotes the non-local Dirac operator defined by

$$D(-\mathrm{i}\partial)\equiv\mathrm{i}\gamma_\mu\partial_\mu+\mathrm{i}\sqrt{M(-\mathrm{i}\partial)}U^{\gamma_5}(x)\sqrt{M(-\mathrm{i}\partial)}.\quad(6)$$

Since the Fermi constant $G_{\rm F}$ is very small, we can expand the exponent in (5) in powers of $G_{\rm F}$ and keep the lowest order only. Thus, the $\Delta S=0$ EWχL can be derived as

$$\mathcal{L}_{\rm eff}^{\Delta S=0}=\int\mathcal{D}\psi\mathcal{D}\psi^\dagger\mathcal{H}_W^{\Delta S=0}\exp\int {\rm d}^4z\ \psi^\dagger(z)D\psi(z). \quad (7)$$

The vacuum expectation value (VEV) of the four-fermion operators in the effective weak Hamiltonian can be calculated as

$$\begin{aligned}&\langle\mathcal{Q}^i(x)\rangle\\&=-\frac{1}{\mathcal{Z}}\int{\rm d}^4y\delta^4(x-y)\frac{\delta}{\delta J_1^i(x)}\frac{\delta}{\delta J_2^i(y)}\int\mathcal{D}\psi\mathcal{D}\psi^\dagger\\&\quad\times\exp\int{\rm d}^4z\ \psi^\dagger(z)\\&\qquad(D+J_1^i(z)\Gamma_1^i+J_2^i(z)\Gamma_2^i)\psi(z)\Big|_{J_1=J_2=0}\\&={\rm tr}_{c,\gamma,f}\left[\langle x|D^{-1}\Gamma_1^{(i)}|x\rangle\langle x|D^{-1}\Gamma_2^{(i)}|x\rangle\right]\\&\quad-{\rm tr}_{c,\gamma,f}\left[\langle x|D^{-1}\Gamma_1^{(i)}|x\rangle\right]{\rm tr}_{c,\gamma,f}\left[\langle x|D^{-1}\Gamma_2^{(i)}|x\rangle\right],\end{aligned} \quad (8)$$

where ${\rm tr}_{c,\gamma,f}$ means the trace over color, spin, and flavor space, respectively. The last two lines in (8) correspond to the unfactorized and factorized quark loops, respectively. $\langle x|(D)^{-1}\Gamma_{1,2}^{(i)}|x\rangle$ can be easily calculated as

$$\begin{aligned}&\langle x|D^{-1}\Gamma_l^{(i)}|x\rangle\\&=\int\frac{{\rm d}^4k}{(2\pi)^4}\ \frac{1}{D^\dagger(\partial+{\rm i}k)D(\partial+{\rm i}k)}D^\dagger(\partial+{\rm i}k)\Gamma_l^{(i)}\ .\end{aligned} \quad (9)$$

The denominator of (9) can be expanded to order $\mathcal{O}(k^2)$ as follows:

$$\begin{aligned}&D^\dagger(\partial+{\rm i}k)D(\partial+{\rm i}k)\\&=-\partial^2+k^2-2{\rm i}k\cdot\partial+M^2-M\left(\gamma_\mu\partial_\mu U^{\gamma_5}\right)\\&\quad-2{\rm i}M\tilde{M}'k_\mu\left[2\partial_\mu+U^{-\gamma_5}\left(\partial_\mu U^{\gamma_5}\right)\right]\\&\quad-M\tilde{M}'\left[2\partial^2+U^{-\gamma_5}\left(\partial^2U^{\gamma_5}\right)+2U^{-\gamma_5}\left(\partial_\mu U^{\gamma_5}\right)\partial_\mu\right]\\&\quad-2M\tilde{M}''k_\mu k_\nu\\&\quad\times\left[2\partial_\mu\partial_\nu+U^{-\gamma_5}\left(\partial_\mu\partial_\nu U^{\gamma_5}\right)+2U^{-\gamma_5}\left(\partial_\mu U^{\gamma_5}\right)\partial_\nu\right]\\&\quad-2\tilde{M}'^2k_\mu k_\nu\left[\left(\partial_\mu U^{-\gamma_5}\right)\left(\partial_\nu U^{\gamma_5}\right)\right.\\&\quad\left.+U^{-\gamma_5}\left(\partial_\mu\partial_\nu U^{\gamma_5}\right)+2U^{-\gamma_5}\left(\partial_\mu U^{\gamma_5}\right)\partial_\nu+2\partial_\mu\partial_\nu\right]\\&\quad+{\rm i}\tilde{M}'k_\mu\left[\left(\partial_\mu\,\gamma\cdot\partial U^{\gamma_5}\right)+2\left(\gamma\cdot\partial U^{\gamma_5}\right)\partial_\mu\right]+\mathcal{O}\left(\partial^3\right).\end{aligned} \quad (10)$$

The numerator reads

$$D^\dagger\left(\partial+{\rm i}k\right)={\rm i}\gamma_\mu\left(\partial_\mu+{\rm i}k_\mu\right)-{\rm i}B, \quad (11)$$

where

$$\begin{aligned}B&=M(k)U^{-\gamma_5}-{\rm i}\tilde{M}'k\cdot\left(\partial U^{-\gamma_5}\right)\\&\quad-\left(M''-\frac{\tilde{M}'^2}{2M}\right)k_\alpha k_\beta\left(\partial_\alpha\partial_\beta U^{-\gamma_5}\right)-\frac{\tilde{M}'}{2}.\end{aligned} \quad (12)$$

Therefore, we have

$$\begin{aligned}&\langle x|D^{-1}\Gamma_l^{(i)}|x\rangle\\&=\int\frac{{\rm d}^4k}{(2\pi)^4}\ \frac{1}{k^2+M^2(k)-A}({\rm i}\gamma_\mu k_\mu-B)({\rm i}\Gamma_l^{(i)})\\&=\sum_{n=0}^{\infty}\int\frac{{\rm d}^4k}{(2\pi)^4}\ \frac{1}{k^2+M^2(k)}\left(\frac{1}{k^2+M^2(k)}A\right)^n\\&\quad\times({\rm i}\gamma_\mu k_\mu-B)({\rm i}\Gamma_l^{(i)}),\end{aligned} \quad (13)$$

where the form of A can be extracted from (10). The expansion of (13) yields the terms to order $\mathcal{O}(\partial^2)$:

$$\begin{aligned}&\langle x|D^{-1}\Gamma_l^{(i)}|x\rangle\\&=\left(\mathcal{I}_1U^{-\gamma_5}+\mathcal{I}_2U^{-\gamma_5}\left(\partial_\alpha U^{\gamma_5}\right)\gamma_\alpha+\mathcal{I}_3\left(\partial^2U^{-\gamma_5}\right)\right.\\&\quad\left.+\mathcal{I}_4U^{-\gamma_5}\left(\partial_\alpha U^{\gamma_5}\right)\left(\partial_\beta U^{\gamma_5}_{5}{}^{\dagger}\right)\gamma_\alpha\gamma_\beta\right){\rm i}\Gamma_l^{(i)}\end{aligned} \quad (14)$$

with the coefficients

$$\mathcal{I}_1=-\int\frac{{\rm d}^4k}{(2\pi)^4}\ \frac{M(k)}{k^2+M^2(k)}=\frac{\langle\overline{\psi}\psi\rangle_{\rm M}}{4N_c}, \quad (15)$$

$$\mathcal{I}_2=\int\frac{{\rm d}^4k}{(2\pi)^4}\ \frac{M^2(k)-\frac{k^2}{2}M(k)\tilde{M}'}{(k^2+M^2(k))^2}, \quad (16)$$

$$\begin{aligned}\mathcal{I}_3=\int\frac{{\rm d}^4k}{(2\pi)^4}&\left[\frac{\frac{1}{4}\tilde{M}''k^2+\frac{1}{2}\tilde{M}'-\frac{\tilde{M}'^2}{8M}k^2}{k^2+M^2(k)}\right.\\&-\frac{M+M^2\tilde{M}'+\frac{k^2}{2}M^2\tilde{M}''+\frac{1}{2}k^2M\tilde{M}'^2+\frac{k^2}{4}\tilde{M}'}{(k^2+M^2(k))^2}\\&\left.+k^2\frac{\frac{1}{2}M+2M^2\tilde{M}'+M^3\tilde{M}'^2}{(k^2+M^2(k))^3}\right],\end{aligned} \quad (17)$$

$$\mathcal{I}_4=\int\frac{{\rm d}^4k}{(2\pi)^4}\ \frac{-M^3+k^2M^2\tilde{M}'}{(k^2+M^2(k))^3}\ . \quad (18)$$

Substituting (14) into (8), taking trace over color and spin spaces, and summing all four-fermion operators, we arrive at the $\Delta S=0$ EWχL to order $\mathcal{O}(\partial^2)$ in terms of the Goldstone boson fields with the LEC determined:

$$\begin{aligned}&\mathcal{L}_{\rm eff}^{\Delta S=0}\\&=\mathcal{N}_1\left(\langle(R_\mu-L_\mu)\lambda_1\rangle\langle(R^\mu+L^\mu)\lambda_1\rangle\right.\\&\quad\left.+\langle(R_\mu-L_\mu)\lambda_2\rangle\langle(R^\mu+L^\mu)\lambda_2\rangle\right)\\&\quad+\mathcal{N}_2\left(\langle(R_\mu-L_\mu)\lambda_4\rangle\langle(R^\mu+L^\mu)\lambda_4\rangle\right.\\&\quad\left.+\langle(R_\mu-L_\mu)\lambda_5\rangle\langle(R^\mu+L^\mu)\lambda_5\rangle\right)\\&\quad+\mathcal{N}_3\langle R_\mu-L_\mu\rangle\langle R^\mu+L^\mu\rangle\\&\quad+\mathcal{N}_4\langle R_\mu-L_\mu\rangle\left\langle(R^\mu+L^\mu)\left(-\frac{I}{3}+\lambda_3+\frac{1}{\sqrt{3}}\lambda_8\right)\right\rangle\end{aligned}$$

$$
\begin{aligned}
&+\mathcal{N}_5\left\langle (R_\mu-L_\mu)\left(-\frac{I}{3}+\lambda_3+\frac{1}{\sqrt{3}}\lambda_8\right)\right\rangle\langle R^\mu+L^\mu\rangle\\
&+\mathcal{N}_6\left\langle (R_\mu-L_\mu)\left(-\frac{I}{3}+\lambda_3+\frac{1}{\sqrt{3}}\lambda_8\right)\right\rangle\\
&\times\left\langle (R^\mu+L^\mu)\left(-\frac{I}{3}+\lambda_3+\frac{1}{\sqrt{3}}\lambda_8\right)\right\rangle\\
&+\mathcal{N}_7\left\langle R_\mu\lambda_1R^\mu\lambda_1-L_\mu\lambda_1L^\mu\lambda_1+R_\mu\lambda_2R^\mu\lambda_2\right.\\
&\left.-L_\mu\lambda_2L^\mu\lambda_2\right\rangle\\
&+\mathcal{N}_8\left\langle R_\mu\lambda_4R^\mu\lambda_4-L_\mu\lambda_4L^\mu\lambda_4+R_\mu\lambda_5R^\mu\lambda_5\right.\\
&\left.-L_\mu\lambda_5L^\mu\lambda_5\right\rangle\\
&+\mathcal{N}_9\left\langle (R_\mu R^\mu-L_\mu L^\mu)\left(\lambda_3+\frac{1}{\sqrt{3}}\lambda_8\right)\right\rangle\\
&+\mathcal{N}_{10}\left\langle R_\mu\left(\lambda_3+\frac{1}{\sqrt{3}}\lambda_8\right)R^\mu\left(\lambda_3+\frac{1}{\sqrt{3}}\lambda_8\right)\right.\\
&\left.-L_\mu\left(\lambda_3+\frac{1}{\sqrt{3}}\lambda_8\right)L^\mu\left(\lambda_3+\frac{1}{\sqrt{3}}\lambda_8\right)\right\rangle \qquad (19)
\end{aligned}
$$

where $R_\mu\equiv \mathrm{i}U\partial_\mu U^\dagger$, $\langle\ldots\rangle$ represents again the trace over flavor space, and $\mathcal{N}_i$ denote the LEC expressed as follows:

$$
\begin{aligned}
&\mathcal{N}_1=2N_c^2\mathcal{I}_2^2\tilde{\alpha}_{11},\quad \mathcal{N}_2=2N_c^2\mathcal{I}_2^2\tilde{\alpha}_{22},\quad \mathcal{N}_3=4N_c^2\mathcal{I}_2^2\tilde{\gamma}_{11},\\
&\mathcal{N}_4=4N_c^2\mathcal{I}_2^2\tilde{\gamma}_{12},\quad \mathcal{N}_5=4N_c^2\mathcal{I}_2^2\tilde{\gamma}_{21},\quad \mathcal{N}_6=4N_c^2\mathcal{I}_2^2\tilde{\gamma}_{22},\\
&\mathcal{N}_7=2N_c\mathcal{I}_2^2\left(\tilde{\alpha}_{11}+2\tilde{\beta}_{11}\right),\\
&\mathcal{N}_8=2N_c\mathcal{I}_2^2\left(\tilde{\alpha}_{22}+2\tilde{\beta}_{22}\right),\\
&\mathcal{N}_9=4N_c\left[4\mathcal{I}_1\mathcal{I}_3\left(\tilde{\gamma}_{12}+\tilde{\gamma}_{21}+2\tilde{\rho}_{12}+2\tilde{\rho}_{21}\right)\right.\\
&\qquad\left.+\mathcal{I}_2^2\left(\tilde{\gamma}_{12}-\tilde{\gamma}_{21}+2\tilde{\rho}_{12}-2\tilde{\rho}_{21}\right)\right],\\
&\mathcal{N}_{10}=4N_c\mathcal{I}_2^2\left(\tilde{\gamma}_{22}+2\tilde{\rho}_{22}\right). \qquad (20)
\end{aligned}
$$

Here, $\tilde{\mathcal{C}}_{ij}$ stand for $\frac{G_{\mathrm{F}}}{\sqrt{2}}\sin\theta_{\mathrm{C}}\cos\theta_{\mathrm{C}}\mathcal{C}_{ij}$ generically, where $\mathcal{C}_{ij}=\alpha,\,\beta,\,\gamma,\,\rho$ in (1). Note that the α_{ij} and γ_{ij} enter in the leading order (LO) Lagrangian, while the β_{ij} and ρ_{ij} appear only in the subleading order in N_c. The numerical evaluation of the LEC will be discussed in the next section.

3 Results and discussion

The large N_c expansion in the context of non-leptonic decays have been discussed already extensively [38–43]. While the large N_c argument works very well in the strong interaction, it does not seem to describe the non-leptonic weak interactions in the leading order (LO) of the large N_c expansion. The strict $1/N_c$ expansion is identical to a naive factorization: There is no mixing in the operators and it leaves only the original four-quark operator which contains the product of two conserved currents. Thus, one has to consider the NLO in the $1/N_c$ expansion. However, if the NLO contribution is large, a problem of its convergence would arise. Moreover, there are various sources of the NLO corrections in the large N_c expansion such as mesonic loop contributions. We are not in a position to take into account all possible NLO corrections in this work. Thus, we will restrict our scheme in the following: First, we will treat the Wilson coefficients in a more practical way, i.e. we will not consider the N_c behavior of the Wilson coefficients. Second, we consider the NLO in the $1/N_c$ expansion at the quark level. It does not mean that these corrections are more important or favorable, compared to other $1/N_c$ corrections such as mesonic loop corrections. We only intend in the present work to compare the LO contribution with the NLO corrections at the quark level. By doing that, we will see that the structure of the $\Delta S=0$ EWχL is rather different from the $\Delta S=1$ EWχL.

We first consider the $\Delta S=0$ EWχL in the LO of N_c, and investigate its behavior with respect to the form factors and the Wilson coefficients. In the large N_c limit, the EWχL becomes

$$
\begin{aligned}
&\mathcal{L}_{\mathrm{eff}}^{\Delta S=0,N_c^2}\\
&=\frac{16\mathcal{I}_2^2N_c^2}{f_\pi^4}\left[2\left(\tilde{\alpha}_{11}\sum_{i=1}^{2}V_\mu^iA^{i\mu}+\tilde{\alpha}_{22}\sum_{i=4}^{5}V_\mu^iA^{i\mu}\right)\right.\\
&+9\tilde{\gamma}_{11}A_\mu^0V^{0\mu}\\
&+3\tilde{\gamma}_{12}\left(-V_\mu^0+2V_\mu^3+\frac{2}{\sqrt{3}}V_\mu^8\right)A^{0\mu}\\
&+3\tilde{\gamma}_{21}\left(-A_\mu^0+2A_\mu^3+\frac{2}{\sqrt{3}}A_\mu^8\right)V^{0\mu}\\
&+\tilde{\gamma}_{22}\left(-V_\mu^0+2V_\mu^3+\frac{2}{\sqrt{3}}V_\mu^8\right)\\
&\left.\times\left(-A^{0\mu}+2A^{3\mu}+\frac{2}{\sqrt{3}}A^{8\mu}\right)\right], \qquad (21)
\end{aligned}
$$

where V_μ^a and A_μ^a are the vector and axial-vector currents, respectively, defined as

$$
V_\mu^a=\frac{f_\pi^2}{2}\left\langle T^a(R_\mu+L_\mu)\right\rangle,\quad A_\mu^a=\frac{f_\pi^2}{2}\left\langle T^a(R_\mu-L_\mu)\right\rangle. \qquad (22)
$$

T^a is the generator of the $U_f(3)$, $T^a=\left(\frac{1}{3},\frac{\lambda^1}{2},\ldots,\frac{\lambda^8}{2}\right)$.

The EWχL given in (21) has one caveat: In the large N_c limit the four-quark operators turn out to be products of two conserved currents, i.e. the vector and the axial-vector currents. However, the presence of the non-local interaction between quarks and Goldstone bosons, which arises from the momentum-dependent quark mass, breaks the gauge invariance, so that the currents are not conserved. Reference [36] discussed a method of how to avoid this problem. The conserved currents in Euclidean space with the non-local interactions can be derived by gauging the partition function. The pion decay constant f_π^2 can be successfully reproduced by using the modified axial-vector current in the following matrix elements:

$$
\left\langle 0\left|A_\mu^a(x)\right|\pi^b(p)\right\rangle=\mathrm{i}f_\pi p_\mu \mathrm{e}^{\mathrm{i}p\cdot x}\delta^{ab}, \qquad (23)
$$

which indicates that the Takahashi–Ward identity of PCAC is well satisfied with the modified conserved axial-vector current. If we use the usual currents such as $A^a_\mu = \bar\psi\gamma_\mu\gamma_5\lambda^a\psi$, we would end up with the Pagels–Stokar expression for f_π^2:

$$f_\pi^2(\mathrm{PS}) = 4N_c \int \frac{\mathrm{d}^4k}{(2\pi)^4}\frac{M^2 - \frac{1}{4}MM'k}{(k^2+M^2)^2}. \tag{24}$$

Thus, one has to consider the modified conserved currents in (21). However, if we use the $f_\pi^2(\mathrm{PS})$ for the normalization of the effective chiral Lagrangian for convenience, we need not introduce them in (21), since we derive the same results as we use the modified conserved currents. Thus, the prefactor $16\mathcal{I}_2^2N_c^2$ in (21) turns out to be f_π^4.

Moreover, in the strict large N_c limit, the original Cabbibo and Weinberg–Salam Lagrangians need not any renormalization, i.e. the terms with $\alpha_{11}, \alpha_{22}, \gamma_{21}$, and γ_{22} survive. Thus, the $\Delta S = 0$ EWχL in the large N_c limit becomes

$$\begin{aligned}\mathcal{L}_{\mathrm{eff}}^{\Delta S=0,N_c^2} &= \sqrt{2}G_{\mathrm{F}}\Bigg\{\cos^2\theta_{\mathrm{C}}\sum_{i=1}^{2}V^i_\mu A^{i\mu} + \sin^2\theta_{\mathrm{C}}\sum_{i=4}^{5}V^i_\mu A^{i\mu} \\ &- \left(\cos 2\theta_{\mathrm{W}}\left(V^3_\mu + \frac{1}{\sqrt{3}}V^8_\mu\right) - \frac{1}{2}V^0_\mu\right) \\ &\times\left(\frac{1}{2}A^{0\mu} - A^{3\mu} - \frac{1}{\sqrt{3}}A^{8\mu}\right)\Bigg\}. \end{aligned}\tag{25}$$

If we take the limit $\theta_{\mathrm{C}} \to 0$, (25) becomes identical to that in [25]. However, if we take a more practical point of view about the large N_c behavior of the anomalous dimensions [36], we get the $\Delta S = 0$ EWχL in the LO of N_c:

$$\begin{aligned}\mathcal{L}_{\mathrm{eff}}^{\Delta S=0,N_c^2} &= 2\left(\tilde\alpha_{11}\sum_{i=1}^{2}V^i_\mu A^{i\mu} + \tilde\alpha_{22}\sum_{i=4}^{5}V^i_\mu A^{i\mu}\right) + 9\tilde\gamma_{11}A^0_\mu V^{0\mu} \\ &+3\tilde\gamma_{12}\left(-V^0_\mu + 2V^3_\mu + \frac{2}{\sqrt{3}}V^8_\mu\right)A^{0\mu} \\ &+3\tilde\gamma_{21}\left(-A^0_\mu + 2A^3_\mu + \frac{2}{\sqrt{3}}A^8_\mu\right)V^{0\mu} \\ &+\tilde\gamma_{22}\left(-V^0_\mu + 2V^3_\mu + \frac{2}{\sqrt{3}}V^8_\mu\right) \\ &\times\left(-A^{0\mu} + 2A^{3\mu} + \frac{2}{\sqrt{3}}A^{8\mu}\right). \end{aligned}\tag{26}$$

We are now in a position to discuss the LEC in (19), which consist of the Wilson coefficients, the dynamic factors $\mathcal{I}_i$, Cabbibo and Weinberg angles, and the Fermi constant G_{F}, among which the $\mathcal{I}_i$ characterize the important feature of the present approach. As shown in (15), the dynamic factor $\mathcal{I}_1$ is identified as the quark condensate. Reference [36] discussed the dependence of the quark condensate on the M_0, where the zero-mode and dipole-type form factors show a similar dependence, while the Gaussian type brings down the quark condensate noticeably. The $\mathcal{I}_2$ is identical to the Pagels–Stokar pion decay constant given in (24), which is approximately 20% smaller than the correct f_π^2 [35,44,45]. The dynamic factor $\mathcal{I}_3$ is plotted as a function of M_0 in Fig. 1. As in the case of $\mathcal{I}_1$, the Gaussian-type form factor gives the smallest value. It is interesting to compare the present results with those in the case of the $\Delta S = 1$ EWχL [36] for which the quark condensate and $\mathcal{I}_3$-like terms arise from the QCD and electroweak penguin operators, so that they appear in the LO of the N_c expansion ($\mathcal{O}(N_c^2)$). However, they are found here in the NLO ($\mathcal{O}(N_c)$) at the quark level.

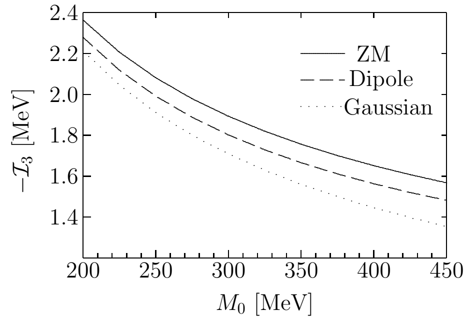

Fig. 1. The dynamic factor $\mathcal{I}_3$ as a function of M_0. The solid curve is drawn with the zero-mode form factor, while the dashed one depicts the dipole-type form factor. The dotted one is for the Gaussian form factor

Taking into account the current conservation properly, we can express the LEC in terms of the pion decay constant f_π, the quark condensate $\langle\bar\psi\psi\rangle$, $\mathcal{I}_3$, and the Wilson coefficients:

$$\begin{aligned}\mathcal{N}_1 &= \frac{f_\pi^4}{8}\tilde\alpha_{11}, \quad \mathcal{N}_2 = \frac{f_\pi^4}{8}\tilde\alpha_{22}, \quad \mathcal{N}_3 = \frac{f_\pi^4}{4}\tilde\gamma_{11}, \\ \mathcal{N}_4 &= \frac{f_\pi^4}{4}\tilde\gamma_{12}, \quad \mathcal{N}_5 = \frac{f_\pi^4}{4}\tilde\gamma_{21}, \quad \mathcal{N}_6 = \frac{f_\pi^4}{4}\tilde\gamma_{22}, \\ \mathcal{N}_7 &= \frac{f_\pi^4}{8N_c}\left(\tilde\alpha_{11} + 2\tilde\beta_{11}\right), \quad \mathcal{N}_8 = \frac{f_\pi^4}{8N_c}\left(\tilde\alpha_{22} + 2\tilde\beta_{22}\right), \\ \mathcal{N}_9 &= 4\langle\overline{\psi}\psi\rangle_M\mathcal{I}_3\left(\tilde\gamma_{12} + \tilde\gamma_{21} + 2\tilde\rho_{12} + 2\tilde\rho_{21}\right) \\ &\quad + \frac{f_\pi^4}{4N_c}\left(\tilde\gamma_{12} - \tilde\gamma_{21} + 2\tilde\rho_{12} - 2\tilde\rho_{21}\right), \\ \mathcal{N}_{10} &= \frac{f_\pi^4}{4N_c}\left(\tilde\gamma_{22} + \tilde\rho_{22}\right)\ . \end{aligned}\tag{27}$$

The corresponding numerical results are listed in Tables 2 and 3. We find that only $\mathcal{N}_9$ depends on the type of form factors.

There is one last remark: We want to mention that there is a matching problem between the scale of the effective weak Hamiltonian and that of the non-local chiral quark model from the instanton vacuum. While the scale of the effective weak Hamiltonian is determined by the renormalization point, which is around 1 GeV, that of the non-local chiral quark model comes from the average size of the instanton,

Table 2. Numerical results of the low-energy constants given in unit of $10^{-5}\mathrm{MeV}^2$. The zero-mode form factor is employed with $M_0 = 350\,\mathrm{MeV}$

	$K=1$	$K=4$	$K=7$
$\mathcal{N}_1$	7.38	8.31	9.34
$\mathcal{N}_2$	0.39	0.44	0.50
$\mathcal{N}_3$	−0.02	0.51	1.07
$\mathcal{N}_4$	0.01	0.00	0.01
$\mathcal{N}_5$	−1.22	−1.40	−1.63
$\mathcal{N}_6$	4.13	4.55	5.11
$\mathcal{N}_7$	2.46	1.26	0.75
$\mathcal{N}_8$	0.13	0.07	0.04
$\mathcal{N}_9$	−7.49	−2.17	0.53
$\mathcal{N}_{10}$	1.38	0.67	0.38

Table 3. Numerical results for $\mathcal{N}_9$ are given in unit of $10^{-5}\mathrm{MeV}^2$ with the dipole-type and the Gaussian-type form factors at $M_0 = 350\,\mathrm{MeV}$. The $\mathcal{N}_9$ value for the zero-mode form factor is given in Table 2

Form factor	$K=1$	$K=4$	$K=7$
Dipole	−6.45	−1.78	0.61
Gaussian	−2.81	−0.42	0.90

i.e. $1/\rho \simeq 600\,\mathrm{MeV}$. Strictly speaking, one has to match these two different scales [46]. However, we will not consider this problem here, since it is a rather delicate one and requires a more cautious investigation.

4 Summary and conclusions

In the present work, we concentrated on deriving the $\Delta S = 0$ effective weak Lagrangian incorporating the effective weak Hamiltonian [10]. Based on the non-local chiral quark model from the instanton vacuum, we obtained the $\Delta S = 0$ parity-violating effective weak chiral Lagrangian with the low-energy constants of the Gasser–Leutwyler type determined. The dependence of the low-energy constants on the dynamic quark mass M_0 and on the type of form factors was studied.

The effects of the strong interaction were introduced according to the two different origins: The effect of non-perturbative QCD which is implemented in the non-local chiral quark model from the instanton vaccum, and the Wilson coefficients which encode the effect of perturbative gluons [10]. In contrast with the $\Delta S = 1$ effective weak chiral Lagrangian [35,36], the factorized quark loops in the integration over the quark field yield the LO terms, while the unfactorized quark loops do the NLO terms. We have determined the low-energy constants consisting of the Wilson coefficients and dynamical quantities such as the pion decay constant and chiral condensate. We have estimated the strong enhancement effects in the LO of the $1/N_c$ expansion. When it is neglected, our result turns out to be equivalent with the effective weak chiral Lagrangian used by [25].

The $\Delta S = 0$ effective weak chiral Lagrangian in the present work can be utilized to various strangeness–conserving weak hadronic processes. For example, one can derive the weak meson coupling constants such as h^1_π. One can also study the parity-violating non–leptonic weak interactions of mesons such as $\eta \to \pi^+\pi^-$ or $\eta \to 2\pi^0$ of which the upper bound of the decay modes are experimentally known only [47].

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